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A simple-shear construction from Thomson & Tait (1867)

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Abstract — Thomson & Tait (*Treatise on Natural Philosophy*, Vol. 1, 1867) described the kinematics of simple shear in terms of a construction. This construction is redescribed in a geological context. It provides a simple method of determining the orientation and amount of strain in a shear zone, and is an excellent aid to teaching the geometry and equations of simple shear.

INTRODUCTION

THE GEOMETRY and kinematics of simple shear have become well-established subjects in structural-geology teaching. They also form the theoretical base for strain analyses of natural shear zones of all scales, which are particularly topical at present. The most complete descriptions of the geometry of simple shear in the geological literature, are given by Ramsay & Graham (1970) and Ramsay (1980).

The kinematics of strain in two and three dimensions were set out succinctly in 1867 in the classic text book of mechanics and dynamics by Thomson & Tait. (Sir William Thomson became Lord Kelvin in 1892, and is generally acknowledged as the founder of theoretical and experimental physics). Thomson & Tait (1867, pp. 106–107, 1962) described the geometry of simple shear, set out the equations relating shear strain and principal strains, and presented a simple construction for determination of the principal strain axes in a zone of shear. I have found no account or application of this construction in the geological literature. Its simplicity invites its use in geological teaching and research.

THOMSON & TAIT CONSTRUCTION (1867)

Thomson & Tait's 1867 analysis is presented here with changes only in nomenclature. The geometry is described in terms of a shear displacement across a given zone which may be called a shear zone. Equal-area plane strain is assumed, so the following analysis is two dimensional. Identification of two sets of lines of unaltered length is the key to the Thomson & Tait construction: these would now be termed the lines of no finite longitudinal strain.

A state of simple shear is illustrated in Fig. 1(a). Point P is displaced to P′ across a zone of unit width, so PP′ is the shear strain γ. Bisect PP′ in N and drop a perpendicular across the shear zone to O (Fig. 1b). OP′ is thus one line of no finite longitudinal strain. Draw a circle centered at O through P and P′ (Fig. 1c). OB is the other line of no finite longitudinal strain. Thomson & Tait (1867, p. 107) stated that AP′ was the direction of elongation strain and BP′ the direction of shortening strain. This can be confirmed in Fig. 1(c) by noting that angle P′AB is half angle P′OB which is the angle between the two lines of no finite longitudinal strain. Thomson & Tait also stated that the value of elongation strain, currently called $(1 + e_1)$, $\lambda^{1/2}$, or X, was given by the ratio AP/AP′. This is equivalent to AP′/BP′, the cotangent of angle P′AB.

The construction described above is drawn in Fig. 1(d) in terms of shear strain γ and angles θ' and $2\theta'$ following Ramsay (1967, 1980). The elongation strain will be written as X and the shortening Z. A set of equations for equal-area simple shear can be presented in terms of the construction in Fig. 1(d).

$$X = \cot \theta' \tag{1}$$

$$\tan 2\theta' = 2/\gamma \tag{2}$$

Substitution of (1) in (2) gives

$$\begin{aligned} \gamma &= X - 1/X \\ &= X - Z. \end{aligned} \tag{3}$$

From (3), X can be written in terms of γ as

$$X = \frac{(\gamma^2 + 4)^{1/2} + \gamma}{2}. \tag{4}$$

Similarly

$$Z = \frac{(\gamma^2 + 4)^{1/2} - \gamma}{2} = 1/X. \tag{5}$$

These five equations may be compared with those given by Ramsay (1980 p. 89) for equal-area strain in simple-shear zones. Ramsay has

$$X^2 = \tfrac{1}{2}[2 + \gamma^2 + \gamma(\gamma^2 + 4)^{1/2}] \tag{6}$$

$$Z^2 = \tfrac{1}{2}[2 + \gamma^2 - \gamma(\gamma^2 + 4)^{1/2}]. \tag{7}$$

These are not written in their simplest form. If (6) is written as

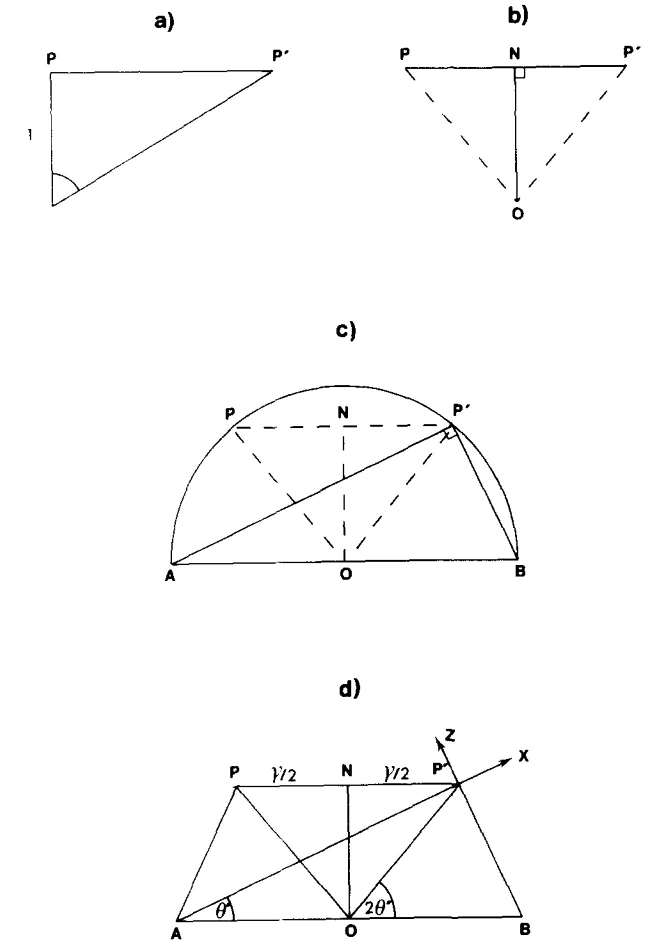


Fig. 1. Thomson & Tait (1867) construction for simple shear. (a) Shear displacement of P to P′ across a zone of unit width. (b) & (c) Stages in the construction. (d) Thomson & Tait construction in modern nomenclature. θ' is the angle of extension X to the shear direction and γ the shear strain.

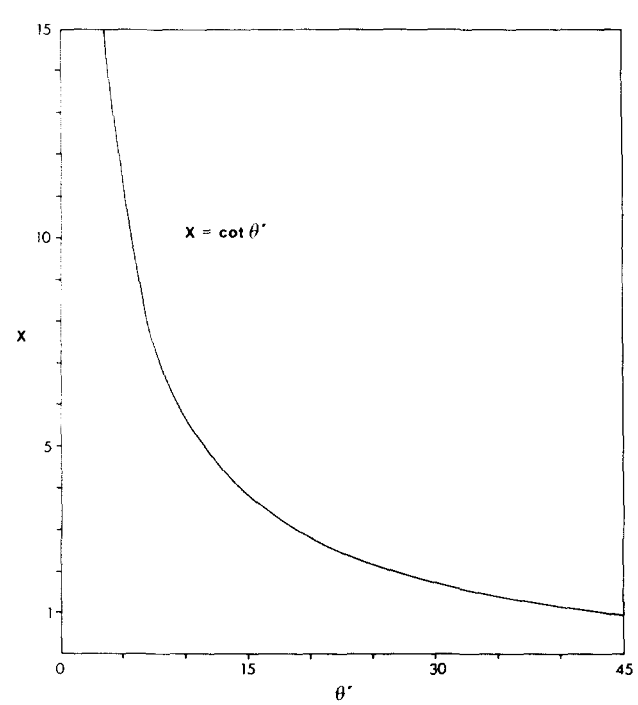


Fig. 2. Graph of $X = \cot \theta'$.

$$X^2 = \tfrac{1}{4}[\gamma^2 + 4 + 2\gamma(\gamma^2 + 4)^{1/2} + \gamma^2],$$

it can be seen that this is a perfect square of equation (4) above.

The simplest equation given by Thomson & Tait is (1): $X = \cot \theta'$. I have not found this used in geological analyses although it can be verified in Fig. 1(d) and also by applying the standard equation of reciprocal quadratic elongation

$$\lambda' = \lambda_1' \cos^2 \theta' + \lambda_2' \sin^2 \theta'$$

to the lines of no finite longitudinal strain. Thus

$$1 = \frac{\cos^2 \theta'}{X^2} + X^2 \sin^2 \theta',$$

the solution to which is $X = \cot \theta'$. This equation is graphed in Fig. 2 so that X can be read directly from θ'.

APPLICATIONS

The Thomson & Tait construction has useful geological applications. It serves as an excellent aid to teaching simple-shear geometry. All the essential equations for equal-area simple shear can be derived from one diagram by elementary geometry. An interesting outcome of this is the simplification of the equations currently in use, and a new equation $X = \cot \theta'$.

The construction can also be applied to natural shear zones which contain deformed markers of any original attitude. The orientation of X, angle θ', is derived from the constructed circle. The value of X is determined by the ratio of two chords or from cotangent θ'. Zones where the shear is approximately uniform are most suitable, such as example 1 below.

Many shear zones, however, are heterogeneous with high γ values in the centre. Such zones could be divided into elements (subzones) of approximately uniform shear, and circles constructed for each subzone, but the circles would be small and the results not accurate. If a sufficient proportion of the shear zone is approximately homogeneous the construction is useful. However, in heterogeneous shear zones in general, θ' and X are the best determined numerically from equations (1)–(5) or Fig. 2, as illustrated by example 2.

Example 1

A brittle–ductile shear zone with approximately uniform strain across it is shown in Fig. 3(a) after Ramsay (1980, fig. 2b). The amount of shear strain is derived from the change in orientation of the veins in the sigmoidal zone from outside the zone where the attitude is taken as undeformed.

The Thomson & Tait construction for the centre of vein V is drawn in Fig. 3(b). The attitudes of X and Z are shown by the lines AP′ and BP′ respectively. The ratio

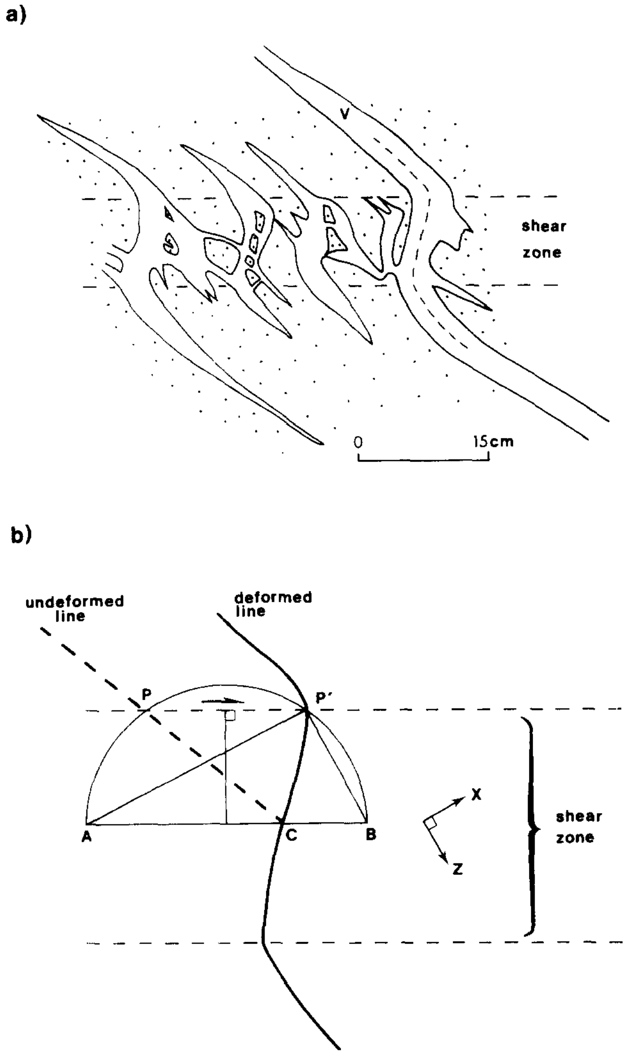
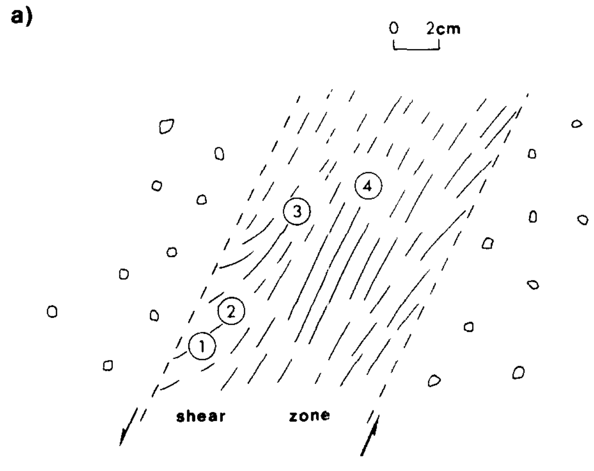


Fig. 3. (a) Brittle-ductile shear zone with en-échelon quartz veins, from Millook Haven, N. Cornwall, England, drawn from Ramsay (1980, fig. 2b). Vein V is selected for study and its centre line marked by a broken line. (b) The Thomson & Tait construction for vein V. The heavy continuous line is an enlargement of the broken line in (a) and the heavy broken line the undeformed vein attitude drawn through C, the centre of the shear zone. The shear displacement for half the zone is P–P′ and the construction follows Fig. 1. The attitudes of X and Z are drawn parallel to AP′ and BP′.

AP′/BP′ gives a value of 1.93 for X. Angle θ' is measured as 27.8° and its cotangent gives $X = 1.9$.

Example 2

Part of a ductile shear zone with the development of a new fabric is sketched in Fig. 4(a) after Ramsay (1980, fig. 2c). The shear strain is clearly heterogeneous reaching a maximum in the centre of the zone. The metagabbro outside the shear zone is approximately homogeneous



b)

locality	θ'	X
1	45°	1·0
2	27°	2·0
3	10°	5·7
4	1° to 0	5·7 to infinity

Fig. 4. (a) Ductile shear zone in Lewisian metagabbro from Castell Odair, N. Uist, Scotland, drawn from Ramsay (1980, fig. 2c). The shear zone is marked by the development of a schistose fabric from an isotropic texture outside the zone. (b) Table of θ' and X values at localities 1 to 4 in (a).

so the fabric in the zone may be attributed to the simple shear.

The attitude of the schistose fabric has been measured at four positions in Fig. 4(a). Using the equation $X = \cot\theta'$ graphed in Fig. 2, the value of X can be readily determined at these points, as tabulated in Fig. 4(b).

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